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BOUNDARY SIMULATION PARAMETERS FOR UNDEREXPANDED JETS IN A QUIESCENT ATMOSPHERE

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NOTICES

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FOREWORD

The research presented in this report was performed by ARO, Inc. (a subsidiary of Sverdrup and Parcel, Inc.), contract operator of the Arnold Engineering Development Center (AEDC), Air Force Systems Command (AFSC), Arnold Air Force Station, Tennessee, under Contract AF 40(600)-1000, Program Element 65402034, ARO Project No. PT8002. The report was submitted by the author on December 17, 1964.

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ABSTRACT

A summary of some of the parameters now in use to obtain jet boundary simulation in a quiescent atmosphere is presented. The shortcomings of these parameters are depicted by showing jet boundaries calculated to an axial distance of ten nozzle exit radii by an approximate technique. Using this approximate technique as a basis for evaluation, a new parameter is presented which, when used in conjunction with the initial inclination angle of the jet, provides a means of specifying jet operating parameters for the simulation of jet boundaries. Methods for preselecting model jet operating and geometric conditions are also discussed.

CONTENTS

ILLUSTRATIONS

NOMENCLATURE

A/A^*	Ratio of nozzle exit area to nozzle throat area
M	Mach number
p	Static pressure, lb/ft^2
p_t	Total pressure, lb/ft^2
r	Jet radius, ft
x	Distance from nozzle base along nozzle axis, ft
β	$\sqrt{M^2 - 1}$
γ	Ratio of specific heats
δ	Angle between tangent to jet boundary and jet axis, deg
θ_N	Nozzle semi-divergence angle, deg
ν	Prandtl-Meyer angle, deg

SUBSCRIPTS

b	Jet boundary conditions
f	Actual jet conditions
j	Nozzle exit conditions
m	Model jet conditions
n	1, 2, 3, etc. (designating locations along the jet boundary)
∞	Ambient conditions

SECTION I
INTRODUCTION

The use of ground test facilities for the study of jet exhausts often necessitates the use of a jet fluid other than that used in the actual flight article. Simulation parameters must therefore be derived so that the characteristics of the test article are similar to those of the flight vehicle. In Ref. 1 the author has presented a summary of the various simulation parameters used in ground tests of jet propulsion devices. Of the many jet properties, the jet boundary shape is deemed one of the most important characteristics for simulation.

In order to obtain similar boundary shapes, a duplication of the initial inclination angle of the jet is required, of course. In Ref. 2 it was shown that, at low ratios of the jet exit to ambient static pressure, boundaries computed for conditions which provided the same value of the initial inclination angle of the jet compared quite favorably with each other. In the study herein reported, the suitability of using the initial inclination angle as a similarity parameter for boundary shapes was explored for jets expanding into a quiescent atmosphere. Duplication of this parameter is shown to be a necessary but not a sufficient condition to obtain similar boundaries. An additional parameter, first introduced in Ref. 3, has been determined, which, based on an approximate theory for the prediction of boundaries, does provide a method for simulation of the jet boundary. A detailed discussion of this parameter is presented in this paper.

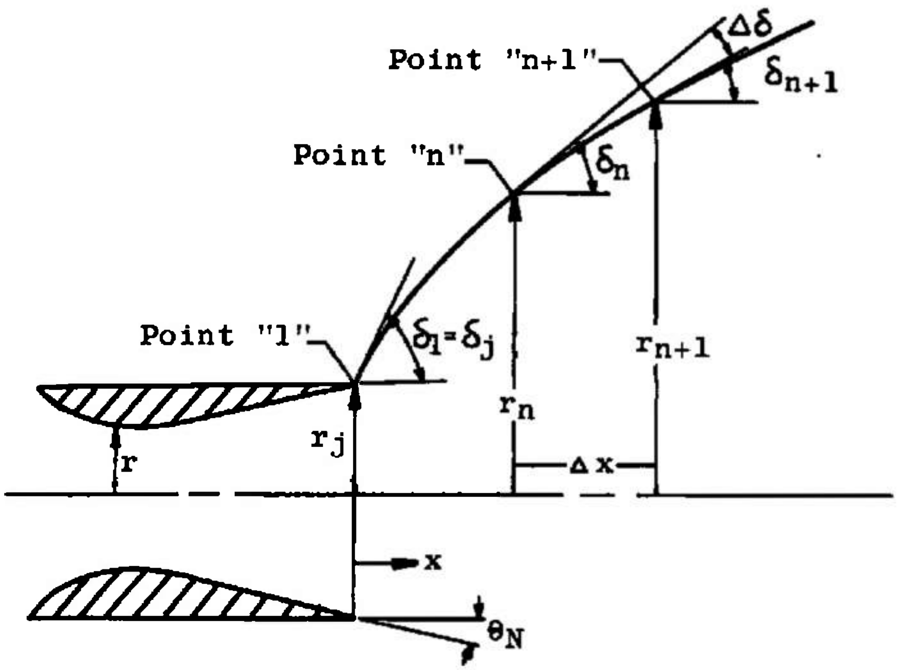
SECTION II
BOUNDARY CALCULATION METHOD

The method used to obtain the jet boundaries given in this report is that presented by Latvala in Ref. 4. The approach is based on the assumption of isentropic, radial flow from the nozzle. As the jet exhausts from the nozzle exit (see sketch below), the initial angle between the tangent to the jet boundary and the jet centerline is given by

$$\delta_j = \theta_N + \nu_1 - \nu_j \tag{1}$$

The value of the nozzle exit angle, θ_N, is fixed by the geometry of the nozzle. The Prandtl-Meyer turning angle of the jet flow, ν_j, is the expansion angle required to accelerate the flow to the Mach number at

the nozzle exit, M_j. The angle ν_1 is that required to expand the flow from the nozzle chamber pressure, p_t, to the ambient pressure, p_∞.



The flow at each spherical surface in the jet stream has associated with it an average Mach number, Prandtl-Meyer angle, and isentropic area ratio. After determining the initial angle of the jet flow from Eq. (1), incremental changes in the jet boundary angle are assumed and a step-by-step method (see Ref. 4) is used to determine the boundary from the following equations:

$$\left[\frac{\frac{r_{n+1}}{r_j}}{\frac{r_n}{r_j}} = \frac{(1+\cos\delta_{n+1})\left(\frac{A}{A^*}\right)_{n+1}}{(1+\cos\delta_n)\left(\frac{A}{A^*}\right)_n}\right]^{1/2} \tag{2}$$

and

$$\frac{\frac{\Delta x}{r_j}}{\frac{r_n}{r_j}} = \left[\frac{\frac{r_{n+1}}{r_j}}{\frac{r_n}{r_j}} - 1\right]\cot\left(\delta_n - \frac{\delta_n - \delta_{n+1}}{2}\right) \tag{3}$$

Boundaries computed by this method are compared with those computed by the method of characteristics in Fig. 1. The agreement between the curves is considered quite acceptable for the purpose of this report.

In order to obtain a boundary using this technique, the following (or equivalent) conditions of the jet flow must be given: (1) the nozzle divergence angle, θ_N, (2) the jet Mach number at the nozzle exit, M_j, (3) the ratio of specific heats of the jet, γ_j, and (4) either the ratio of the jet static pressure at the nozzle exit to the ambient pressure, p_j/p_∞, or the ratio of the ambient pressure to the jet stagnation pressure, p_∞/p_t.

SECTION III
RESULTS USING PREVIOUS ANALYSES

Over the past few years, an ever-increasing number of jet studies have been performed in ground test facilities. Although various simulation parameters have been proposed, very little work has been done on the experimental evaluation of the validity of these parameters. In the following section, jet boundaries are theoretically determined and compared using some of the proposed parameters. The actual jet flow stream used as a basis for the comparisons has the following properties which correspond to a typical jet exhaust at an altitude of approximately 160, 000 feet:

$$\gamma_j = 1.28$$
$$M_j = 3.40$$
$$\theta_N = 10 \text{ deg}$$
$$\delta_j = 60 \text{ deg}$$
$$A/A^* = 8.69$$
$$p_j/p_\infty = 345$$

3.1 CONSTANT INITIAL ANGLE

As can be seen in the previous section, the shape of the jet boundary is dependent upon the same jet flow conditions as the initial inclination angle of the jet; namely, θ_N, M_j, γ_j, and p_j/p_∞. As a first approach, therefore, it would appear that similar boundaries would be obtained if the initial inclination angle of various jet flows was duplicated. As mentioned in the Introduction, use of the initial inclination angle of the jet flow as it emerges from the nozzle has been shown to be an adequate simulation parameter (Ref. 2) for jet exhausts at low pressure ratios ($p_j/p_\infty < 10$). As the following material indicates, however, use of this parameter only, as the pressure ratio is increased (increase in flight altitude), is not a sufficient condition for simulation of the boundary shape.

Boundaries calculated by the method described in Section II for which the initial inclination angle of the jet flow, δ_j, was kept constant at 60 deg are shown in Figs. 2 and 3. In the first example, Fig. 2, the geometric parameters of the nozzle, θ_N and $A/A*$, are kept the same as that of the actual jet while the jet fluid is varied (γ_j variable). In the second example, Fig. 3, the jet fluid and nozzle exit angle were kept the same as the actual jet (γ_j and θ_N constant) while the Mach number of the jet flow, M_j, was varied over a wide range. As is apparent from both examples, keeping the initial angle of the jet flow constant is not a sufficient condition to obtain duplicate jet boundaries.

3.2 JET EXIT FLOW PARAMETER

In Ref. 5, a similarity parameter was derived by considering that the static pressure change caused by a change in the flow direction in the jet must be equal for an actual and simulated jet flow. This requirement leads to the relationship referred to in this report as the jet exit flow parameter which is given as

$$\left(\frac{\gamma_j M_j^2}{\beta_j}\right)_m = \left(\frac{\gamma_j M_j^2}{\beta_j}\right)_f \tag{4}$$

As shown in Ref. 1, this parameter is identical to that derived by Kawamura (Ref. 6) while considering simulation of the jet wave structure. As an additional requirement for simulation, it is specified in Ref. 5 that the ratio of the jet exit to ambient static pressure for the actual and the simulation flows must be equal.

Boundaries have been computed for various jet fluids (γ_j variable) with the jet exit flow parameter, $(\gamma_j M_j^2)/\beta_j$, and the static pressure ratio, p_j/p_∞, kept constant at values corresponding to the actual jet conditions. At each set of operating conditions, an adjustment in jet Mach number is made to keep the jet exit flow parameter constant. The resulting boundaries are presented in Fig. 4. Although simulation of the jet boundary is much improved over that obtained using the initial angle as the simulation parameter, exact duplication of the boundaries is not obtained when using the jet exit flow parameter and the static pressure ratio as simulation parameters.

3.3 COMBINED PARAMETERS

Jet boundaries are shown in Fig. 5 wherein both the jet exit flow parameter and the initial inclination angle of the jet flow are kept constant.

In addition as in Fig. 2 the nozzle exit angle was fixed at 10 deg. As compared to Fig. 2, the jet exit flow parameter rather than the nozzle area ratio defines the required nozzle exit Mach number for the various jet fluids. Although the differences between the actual jet boundary (curve 3 in Fig. 2 and curve 4 in Fig. 5) and the boundaries for other corresponding jet fluids are reduced by using the jet exit flow parameter, the simulation of the boundary is still not acceptable.

In Ref. 1, the suggestion is made that all three of the foregoing parameters (δ_j, $\gamma_j M_j^2/\beta_j$, and p_j/p_∞) be used as similarity parameters. When using a model fluid with a specific heat ratio different from that of the actual jet flow, the jet Mach number is adjusted to keep the jet exit flow parameter the same. Since the pressure ratio is to be kept constant as well as the initial inclination angle, the nozzle exit angle, θ_N, must be adjusted to account for the change in the difference in the Prandtl-Meyer flow angles (see Eq. (1)). A comparison of boundaries obtained under these conditions is shown in Fig. 6. In this case also, the simulation is not adequate.

SECTION IV
RESULTS USING PRESENT ANALYSIS

Inasmuch as none of the previously discussed groups of similarity parameters provided for an exact duplication of the boundary shapes, a study was undertaken to determine a more suitable group of parameters for this purpose. Since the similitude of the initial inclination angle was felt to be imperative, the problem was reduced to an examination of the jet boundary flow after the initial expansion from the nozzle exit had occurred. The problem was thus recognized to be closely related to that of determining approximate jet boundaries.

4.1 SIMULATION PARAMETER

The jet flow at the boundary after the initial expansion can be considered to go through a series of small deflections in order to balance the internal pressure with that of the constant ambient pressure. The first order term of the series expression for supersonic flow through a small deflection

$$\frac{\Delta p_b}{p_\infty} = \frac{\gamma M^2}{\beta} (\Delta \delta_b) \tag{5}$$

can be used to relate the pressure change with the flow deflection. A term similar to the jet exit flow parameter is obtained as the relating parameter between the pressure and angle changes.

This term evaluated at the jet flow conditions just after the flow expansion (subscript 1) gives the following simulation parameter:

$$\left(\frac{\gamma_1 M_1{}^2}{\beta_1}\right)_m = \left(\frac{\gamma_1 M_1{}^2}{\beta_1}\right)_f \tag{6}$$

Boundaries have been computed by the Latvala approximation using Eq. (6) as the simulation parameter as well as keeping the initial inclination angle, δ_j, constant. These boundaries are shown in Fig. 7 for various jet fluids (variable γ_j) with nozzles of constant exit angle and in Fig. 8 for nozzles of varying exit angle but a single jet fluid. In both cases, the comparison of the boundaries is excellent.

4.2 PRESELECTION OF MODEL CONDITIONS

An examination of the freedom of choice of the operating and geometric conditions of a model jet will now be made using δ_j and $\gamma_1 M_1{}^2/\beta_1$ as simulation parameters.

If the model jet fluid is specified, then γ_1 is given since it is assumed that $\gamma_1 = \gamma_j$. The value of the parameter $\gamma_1 M_1{}^2/\beta_1$ in turn fixes the value of M_1 and therefore ν_1 and p_∞/p_t ($p_1 = p_\infty$). The value of δ_j and use of Eq. (1) gives:

$$\nu_1 - \delta_j = \nu_j - \theta_N \tag{7}$$

Thus a freedom in the choice of either ν_j or θ_N still remains at this point. A free choice of ν_j implies a choice of either M_j or p_j/p_∞. A preselection of any one of the conditions θ_N, M_j, or p_j/p_∞ at this point specifies the values of the other two conditions.

As a second example, assume that a nozzle of given geometry (θ_N and A/A^*) is to be used for the model. The only condition that remains to be determined is the ratio of specific heats of the jet. For specific values of δ_j and $\gamma_1 M_1{}^2/\beta_1$ only one particular value of γ_j will satisfy the similarity parameters. Thus, if a fluid with that ratio of specific heats is not available, an adjustment in either θ_N or A/A^* of the nozzle must be made in order to obtain similar boundaries.

In order to illustrate the foregoing, the plot shown in Fig. 9 has been prepared. The actual jet conditions are denoted by the darkened circular symbol. The values of the similarity parameter $\gamma_1 M_1{}^2/\beta_1$ and the initial inclination angle, δ_j, for this example are 10 and 60 deg, respectively. Any set of the four jet and nozzle parameters (θ_N, γ_j, M_j, and p_j/p_∞) shown in this plot will provide the same jet boundary as the actual jet. For example, if CO_2 was to be used as the simulation fluid (γ_j = 1.34), then a choice of any of the parameters p_j/p_∞, M_j, or θ_N exists. Specifying any one of these conditions, however, fixes the values of the other two. If it were desired to operate at the same pressure ratio (p_j/p_∞ = 345) as the actual jet, the model would be designed with a nozzle exit angle of about 6 deg and an exit Mach number of 2.8. If, instead, it was desired to keep the nozzle exit Mach number of the model equal to that of the actual jet, the model would be designed with an exit angle of 18 deg and operated at a pressure ratio of 140. If the nozzle exit angle was to remain 10 deg, the jet Mach number would be 3.0 and the pressure ratio 270. Besides these three sets of conditions, any other set of conditions along the γ_j = 1.34 curve would satisfy the simulation requirements.

4.3 BOUNDARY PREDICTIONS

As mentioned in Section II, the calculation of a jet boundary is dependent upon the following four parameters (or their equivalent); θ_N, M_j, γ_j, and p_j/p_∞. Thus the prediction or pre-calculation of families of jet boundaries covering a representative range of these variables becomes a prodigious task (see Ref. 2). Such an undertaking would be reduced in scope significantly should the foregoing similarity parameters prove successful. As an example, all of the boundaries presented previously which provide an initial inclination angle of 60 deg are shown in Fig. 10. These boundaries, although calculated from greatly varying jet conditions, provide an orderly family of curves using the boundary simulation parameter as a correlation factor. Thus, similar curves for other initial inclination angles would provide a catalogue of boundaries for ready reference.

SECTION V
CONCLUDING REMARKS

The specification of similarity and scaling parameters for jet exhausts has received a fair degree of analytical attention but as yet only a limited degree of experimental verification. Proper simulation

of the jet boundary is in most cases a highly desirable objective. In this paper a parameter is presented which, along with the initial inclination angle of the jet, provides for simulation of the jet boundary based on an approximate boundary calculation technique (see Ref. 4).

Experimental verification of the proposed boundary simulation technique is of course desirable. Such a program would involve testing with various jet fluids to determine whether similar jet boundaries can be obtained using the proposed simulation parameters. Verification of the calculation technique would also be obtained; however, whether or not such verification is obtained is of course incidental to the validity of the simulation technique. Should the technique prove feasible, studies of rocket, turbojet, and ramjet plumes could be performed with much simpler setups than with the use of the actual jet exhausts.

The prediction of jet boundaries for various nozzle geometric and operating conditions would also be simplified if the technique proves to be applicable. Present boundary calculations are dependent upon four independent variables whereas use of the simulation parameters would reduce the specification of a particular boundary to two variables. Thus the pre-calculation of families of boundaries could be made a realistic task.

It should finally be pointed out that in cases where other jet parameters such as jet momentum or jet shock structure are deemed more critical for simulation than the jet boundary, it may not be possible to select jet operating or geometric parameters to satisfy the new simulation parameter as well as the initial inclination angle. The extent of the boundary simulation could, however, be determined in such cases using the present analysis and boundary calculations similar to those presented.

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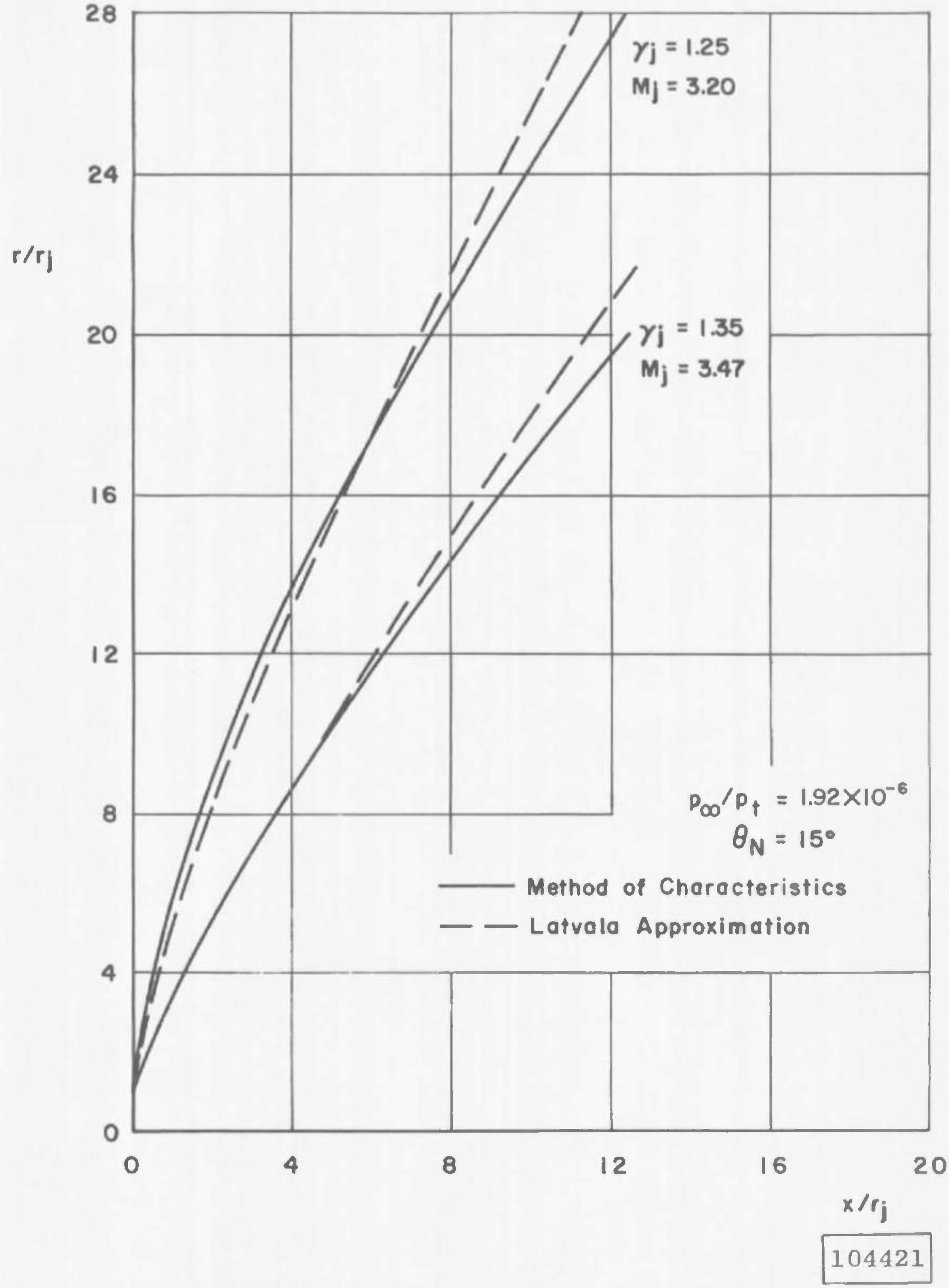

Fig. 1 Comparison of Jet Boundaries Calculated by the Latvala Approximation and the Method of Characteristics

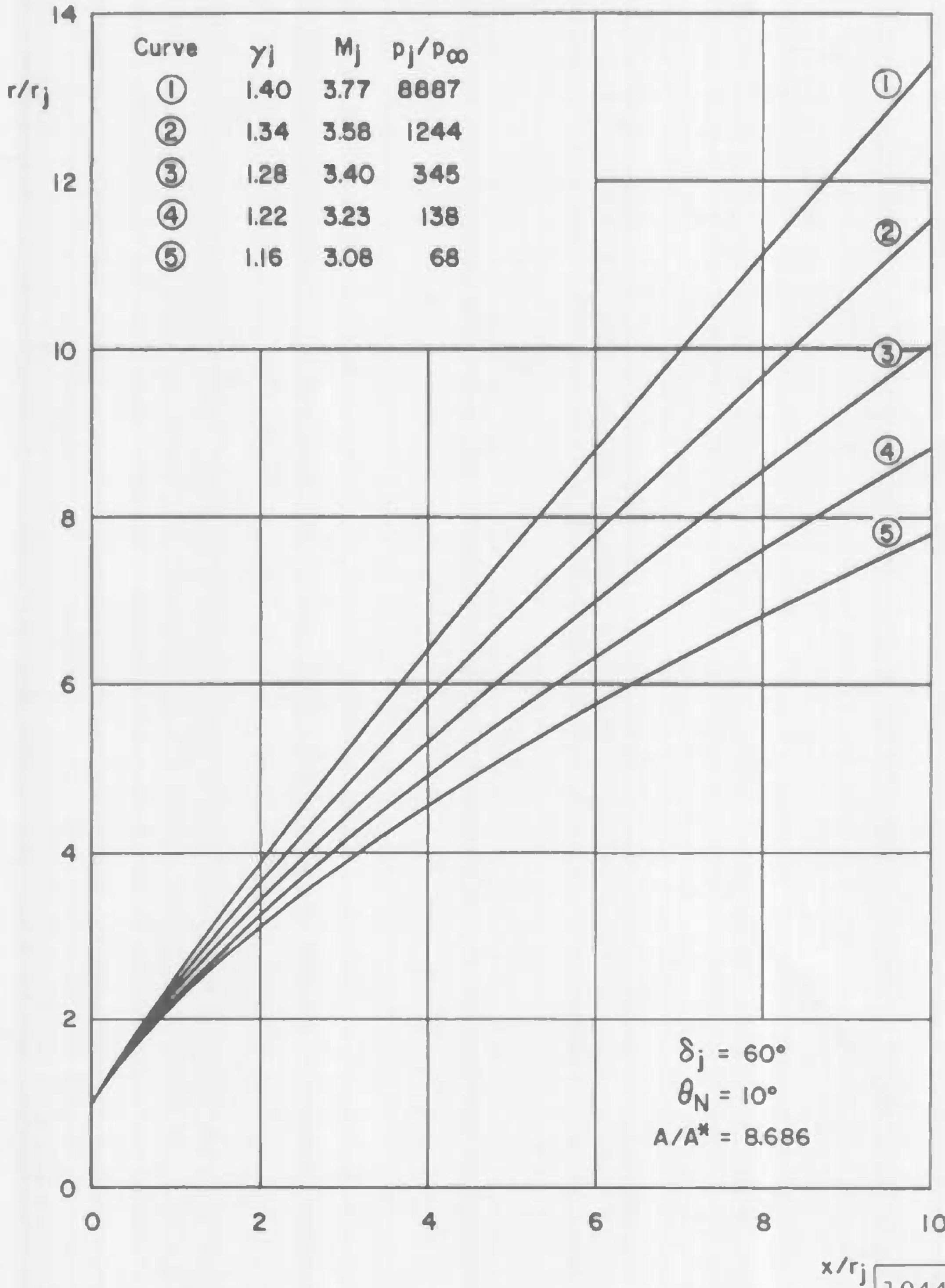

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Fig. 2 Comparison of Jet Boundaries at Constant Initial Inclination Angle; Nozzle Geometry Fixed, Various Jet Fluids

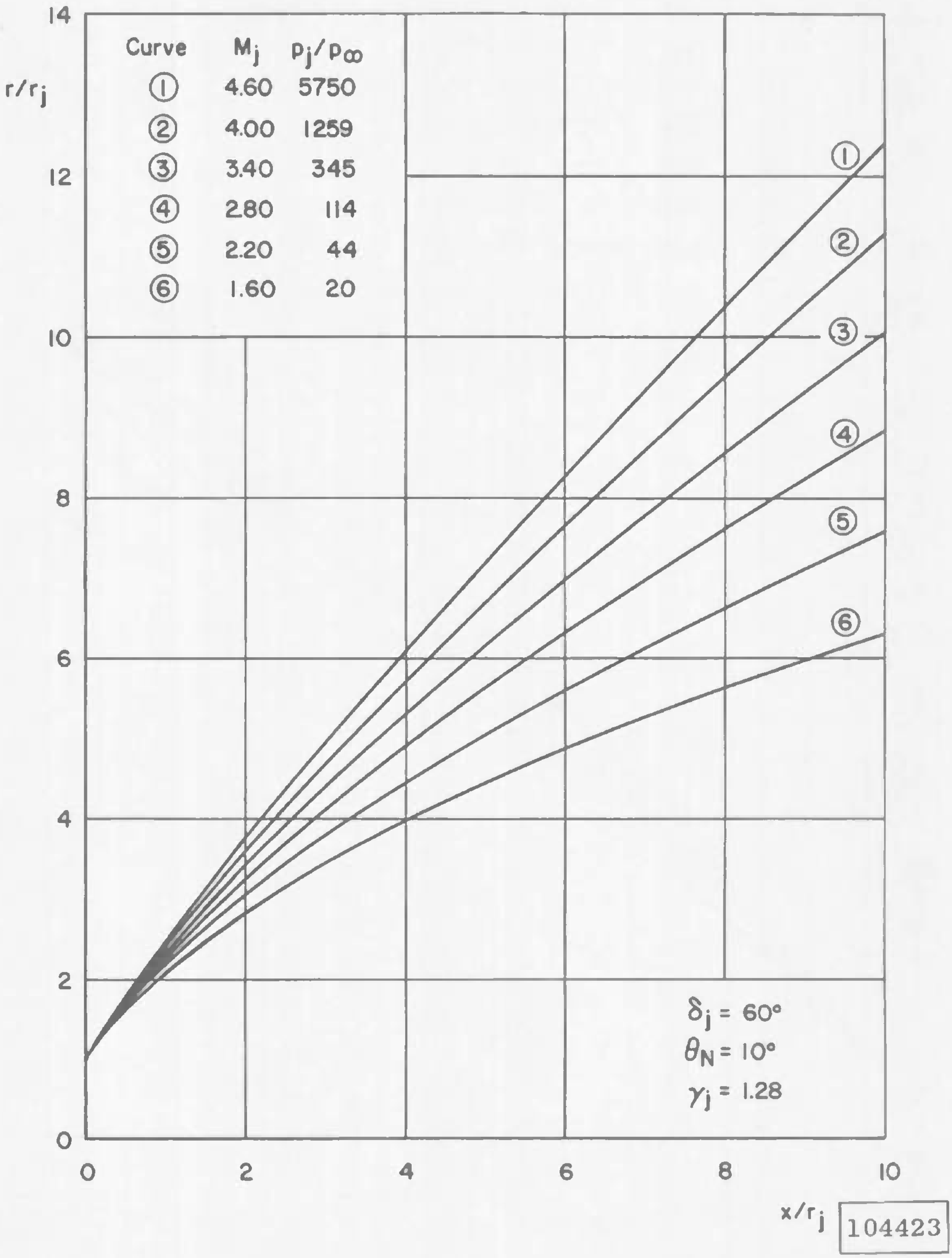

Fig. 3 Comparison of Jet Boundaries at Constant Initial Inclination Angle; Same Jet Fluid, Various Nozzle Mach Numbers

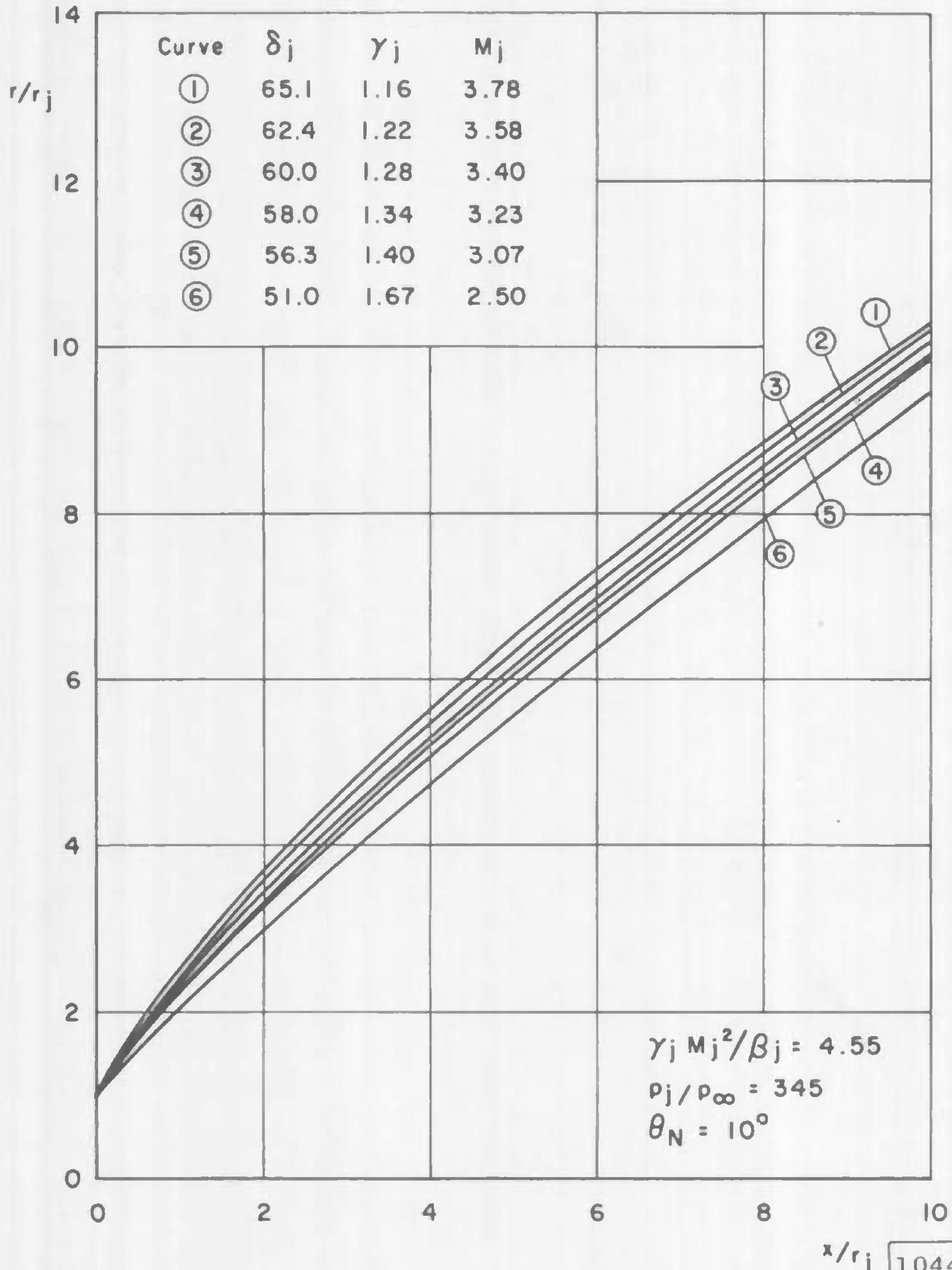

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Fig. 4 Comparison of Jet Boundaries Using Jet Exit Flow Parameter and Jet Pressure Ratio as Simulation Parameters

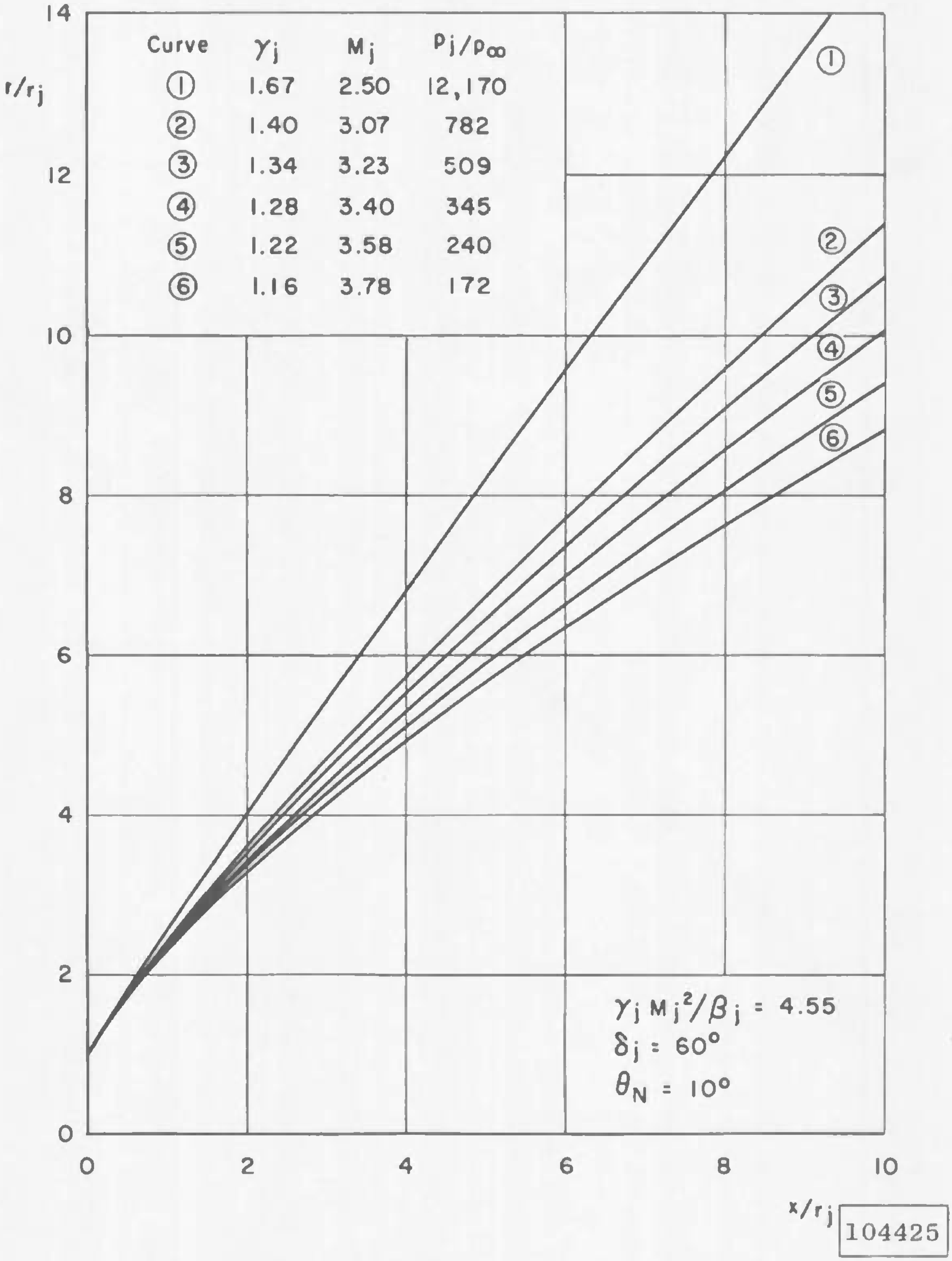

Fig. 5 Comparison of Jet Boundaries Using Jet Exit Flow Parameter, Jet Pressure Ratio, and Initial Inclination Angle as Similarity Parameters

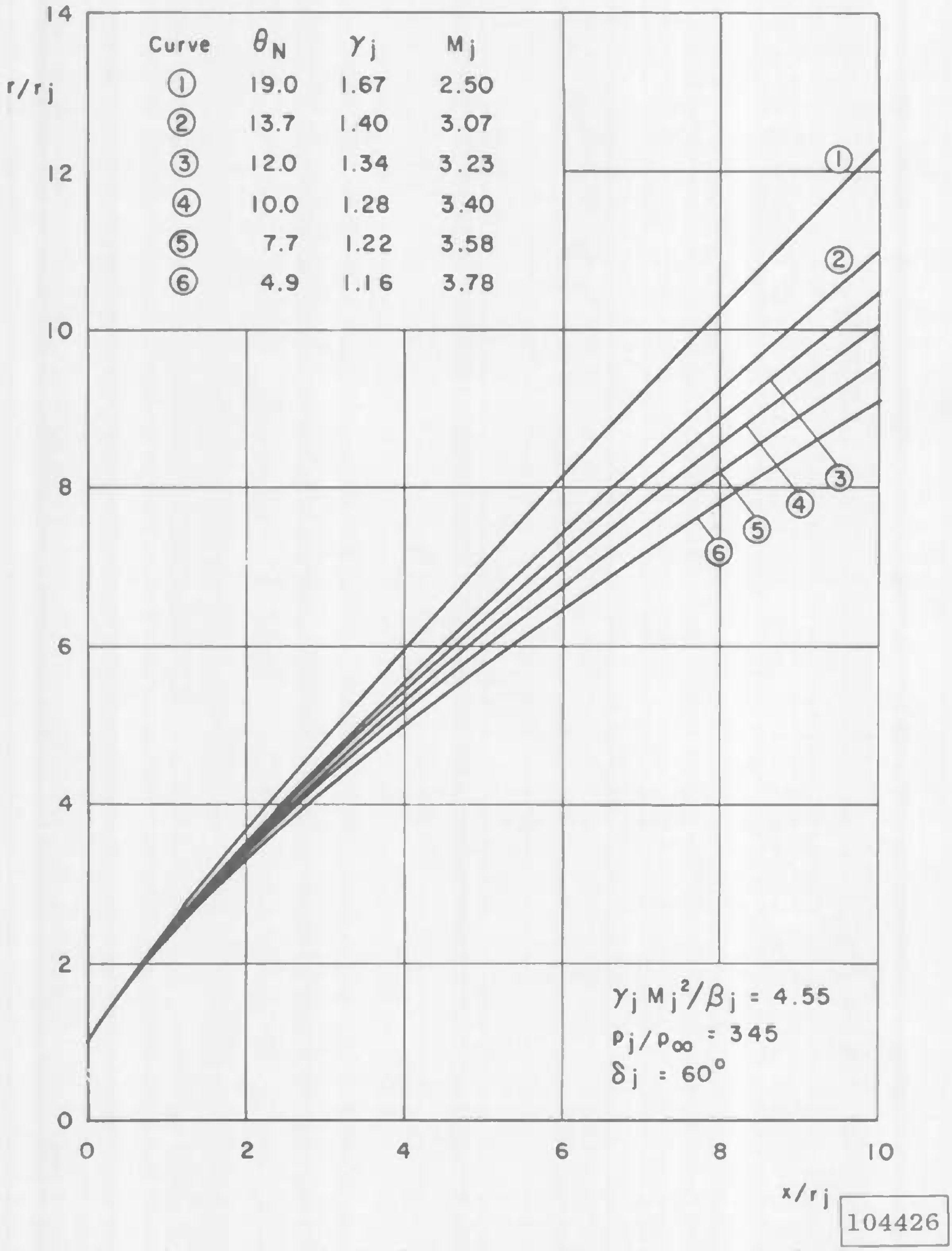

Fig. 6 Comparison of Jet Boundaries Using the Jet Exit Flow Parameter and Initial Inclination Angle as Similarity Parameters; Fixed Nozzle Exit Angle

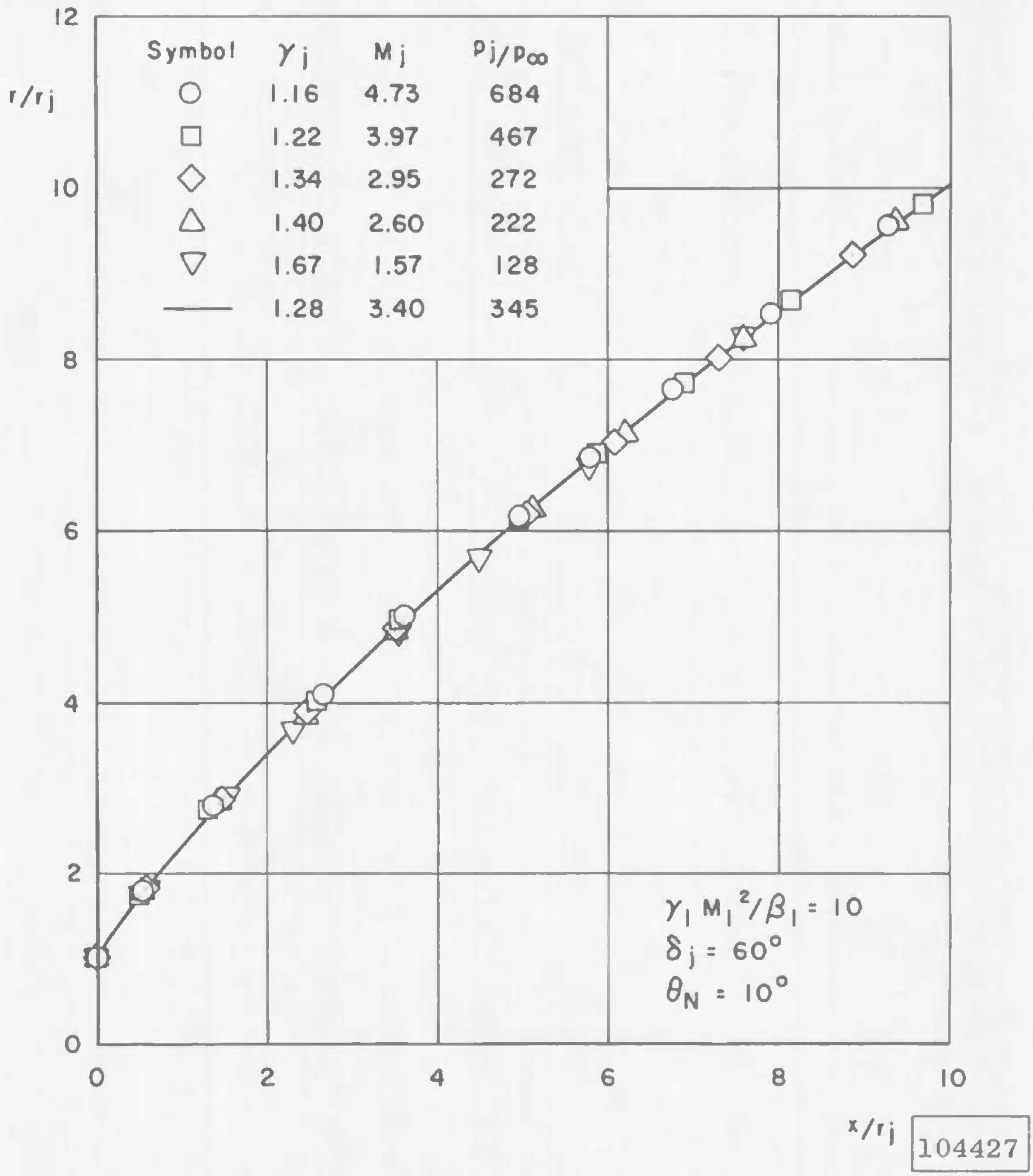

Fig. 7 Comparison of Jet Boundaries Using the New Simulation Parameter and Initial Inclination Angle as Similarity Parameters; Various Jet Fluids

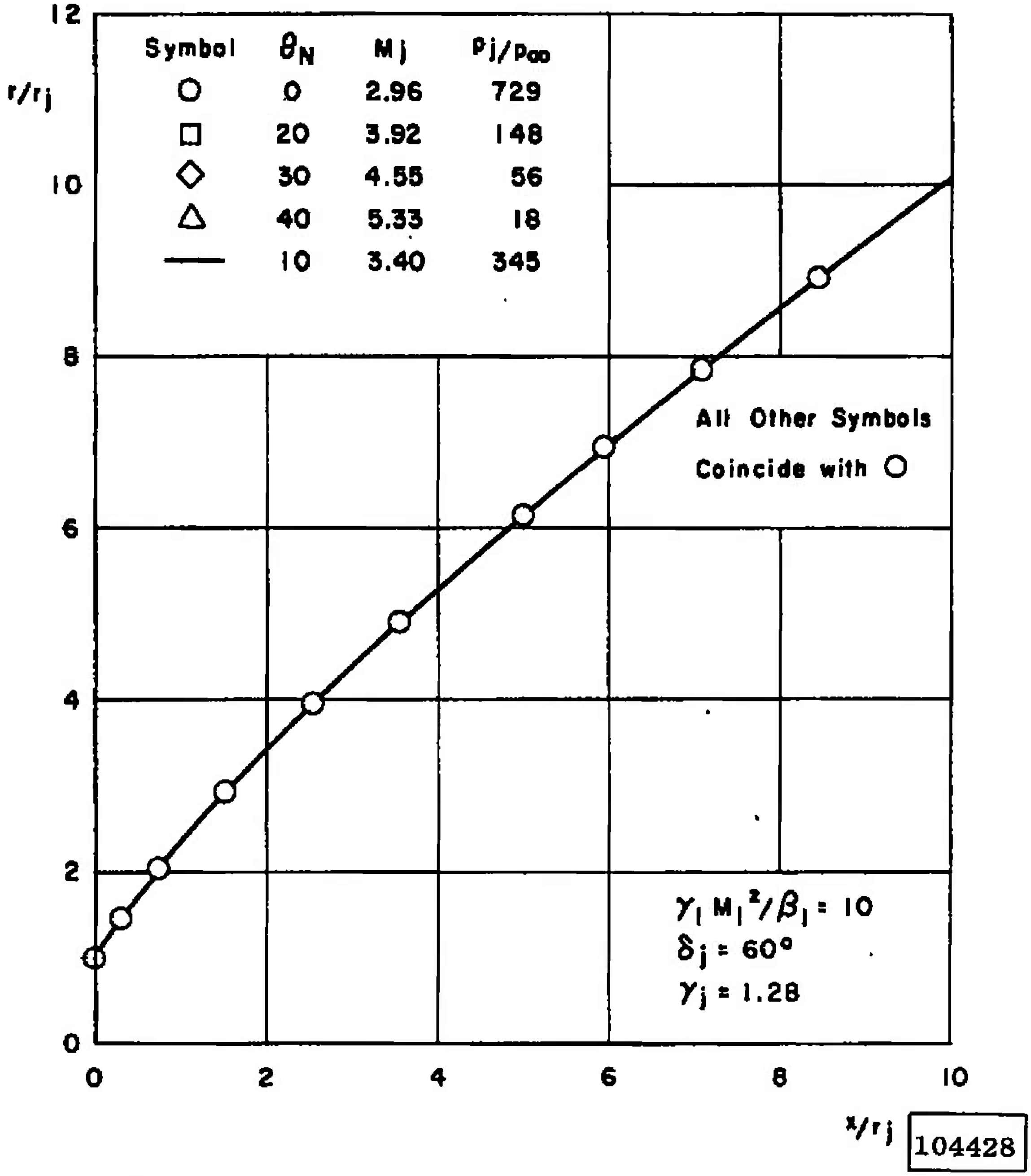

Fig. 8 Comparison of Jet Boundaries Using the New Simulation Parameter and Initial Inclination Angle as Similarity Parameters; Same Jet Fluid

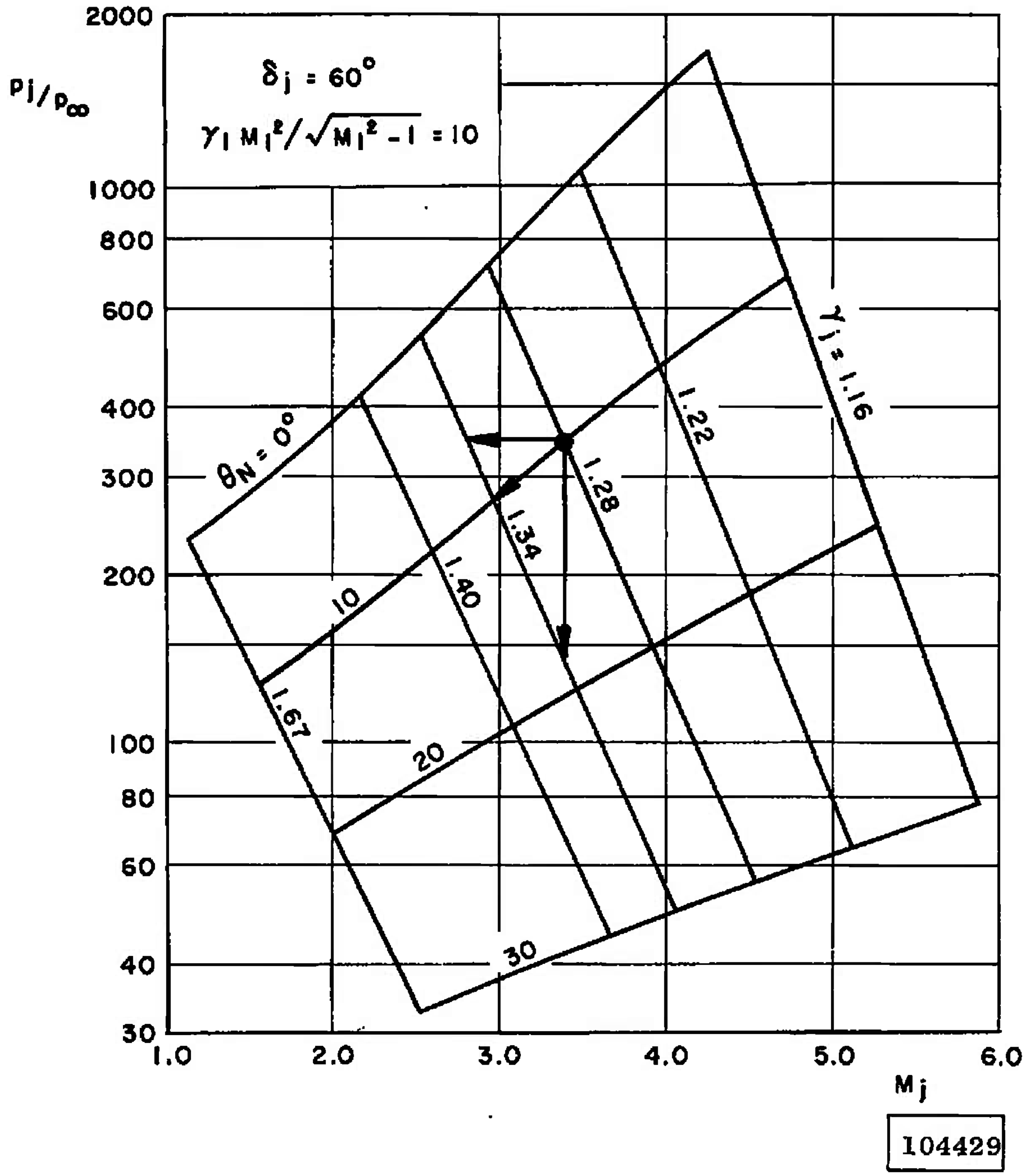

Fig. 9 Map of Nozzle Geometric and Jet Operating Conditions Giving Similar Jet Boundaries

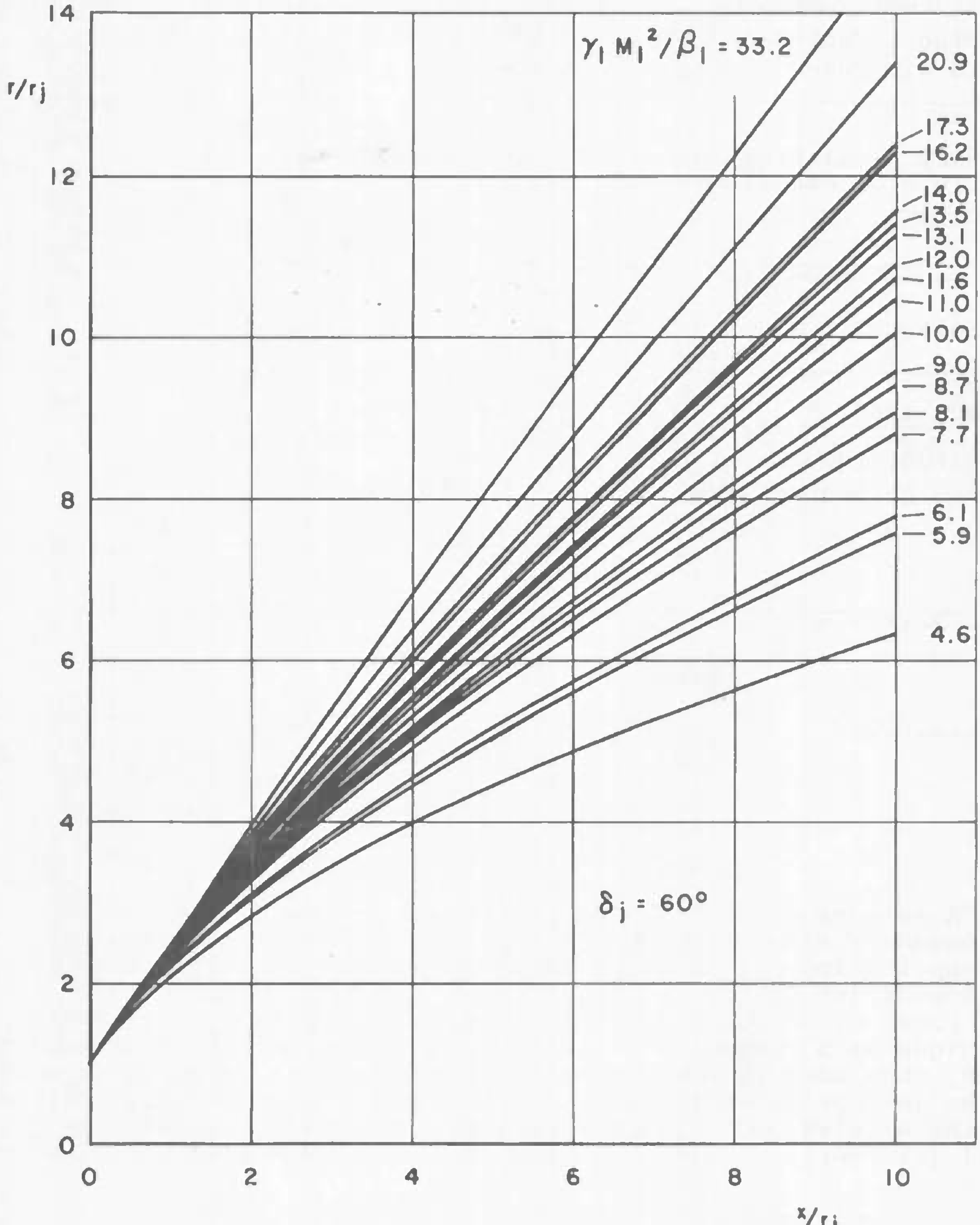

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Fig. 10 Family of Jet Boundaries at a Fixed Initial Inclination Angle and Varying Boundary Simulation Parameter

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